XCS299i Problem Set #3

JUAN RICARDO PEDRAZA ESCOBAR

Machine Learning

Stanford Center for Professional Development

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**Notation.**

In the question, we use to denote the kernel mapping, and use to denote the corresponding kernel matrix generated by some collection .

1.a

is a kernel. Let and denote the corresponding kernel metrics. By definition, the term of the kernel matrix generated by K is

Therefore,

.

Because and are symmetric and also and are valid kernels. Note that both and are positive-semi definite matrices, thus,

So, is a valid kernel. Because the corresponding kernel matrix generated by is symmetric, for any , and positive-semi definite.

1.b

is not a kernel. A counter-example can be constructed as:

Then for any sequence, the corresponding kernel matrices are

We can say that then,

So, K is not positive-semi definite matrix. Hence, is not a valid kernel.

1.c

is a kernel.

For any , let denote the kernel matrix generated by kernel , which is positive-semi definite matrix and symmetric. And the kernel matrix is symmetric. And because for every ∈ ,

Which implies

So K is positive-semi definite matrix and is a valid kernel

1.d

is not a kernel. Let and denote the corresponding kernel matrices. Then, is positive-semi definite matrix. Therefore, for every ∈ ,

Then

So, is not a positive-semi definite matrix.

1.e

is a Kernel. and are kernels, thus such that and .

Therefore,

Now we use and so,

This shows us that we can express K as,

So, is a Kernel.

1.f

is a Kernel, and we have,

So, the generated kernel matrix is symmetric,

Therefore, K is positive-semi definite matrix. So is a valid kernel.

1.g

is a valid kernel. Since is a kernel over × , We are going to see that is a valid kernel on × by showing it can be written as an inner product of feature mapping there exists a feature mapping : → for some l, such that for every x, y ∈ ., then by definition

Therefore is a kernel.

1.h

* is a valid kernel. Taking account, the result from (e) and induction, it can be shown that , where all are kernels. This result permit said that is also a kernel setting all .
* Let denote the coefficients of , and note also that for every . Then by result from part (e) and part (c), is a valid kernel. And by result from part (a) induction, is a valid kernel,